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Short Communication

Assessment of design parameters of a slab track railway system from a dynamic viewpoint

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Abstract

The development of the ballastless slab track, with applications especially on soft soil in combination with loading by high-speed trains, puts several specific engineering demands. One of these is how to provide the required vertical stiffness of the track system. According to the most common approach massive soil improvements are applied. An alternative to this would be to increase the bending stiffness of the slab, e.g. by applying an eccentric reinforcement. Both solutions have consequences for the dynamic track and ground response. In this contribution, the classical model of a beam on elastic half-space subject to a moving load is employed to assess effectiveness of these engineering solutions by analysis of their influence on the generalized dynamic track stiffness. The aim is to minimize the level of slab vibrations, in order to prevent deterioration. The effect of variation of other track properties is also evaluated. It is shown that for high frequencies an increase of the track stiffness is most effective, whereas for low frequencies soil improvement is a better solution. It is further shown that a relatively high track mass generally decreases track vibrations in the relevant frequency domain and that the width of the slab is an important parameter to control the level of track vibrations.

1. Introduction

The relatively new ballast-less slab track or continuous concrete track is applied worldwide on an increasing scale for high-speed railway lines [1]. As an example, more than 4000 km of slab track for high-speed lines are planned in China for the near future [2]. This development is due to the fact that slab track has several advantages over ballasted track. Some structural advantages are: a higher longitudinal and lateral permanent stability, the impossibility of rail buckling and a reduced sensitivity to differential settlements. Operational advantages of slab track are: a lower maintenance (reduction with 70–90% relative to ballasted track [3]), resulting in higher availability and the possibility of longer possession times—which is important for high-speed connections, the prevention of churning up of ballast particles at high speed, and an increase of passenger comfort as well as safety, due to the higher track stability and better alignment. Disadvantages are

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the high initial investment costs and a lower vibration and noise absorption, which may also lead to structural damage at an early stage.

Nearly all designs of current slab-track railways are based on the principle of a relatively flexible continuous or segmented concrete slab on top of a stiff substructure. As many high-speed lines are built in flat delta areas with relatively weak subgrades (Germany, Netherlands, Sweden, Japan, Korea, China), often massive and cost-intensive soil improvements are necessary, especially to increase the critical train velocity (Fig. 1).

According to the German school, based on static highway design, the supporting layer, with a thickness of about 0.3 m, should have a substantial bearing stiffness, with Young's modulus of at least 120 MN/m^2 whereas the embankment below should have a minimum modulus of 60 MN/m^2 [3]. In this case also non-coherent block structures without bending stiffness may be used, and differential settlements are excluded. The applied track designs have reinforcement in the neutral axis in order to control the crack width of the in situ cast concrete in the slab (Figs. 1 and 2).

The Japanese Shinkansen network, with over 1000 km of ballast-less track, consists of short prefabricated slab sections of about 5 m on top of a continuous concrete roadbed as a sublayer [2,3].

These static or quasistatic design principles do not account for the fact that a railway track, with a width in the order of 3 m, or approximately equal to the train width, is essentially different from a road pavement, where the width generally significantly exceeds the vehicle width. Thus, railway tracks may be considered as predominantly one-dimensional (1-D) structures, in which longitudinal bending moments due to train passage are dominant with respect to moments across the track. This is not the case in a road superstructure, which should be considered as a flexible plate, provided that its structure is coherent. Additionally, loading of a railway track is in general symmetric with respect to its axis of symmetry. Therefore, variation in lateral bending moment due to a varying loading position across the track can be neglected, in contrast to that for a road pavement.

Another difference is the fact that running trains may approach or even exceed critical track velocities, whereas this is not the case for road traffic.

On the basis of the above, the traditional slab track design may be stated to be rather conservative, and, due to the generally massive soil improvements, expensive relative to traditional ballasted concepts.

Fig. 1. Different stages of installation of a slab track railway system: (a, b) soil improvement, (c) automated installation of HSL-layer (see Fig. 2; class B5, without reinforcement), (d) installation of duoblock sleepers and central reinforcement, (e) in situ casting of the concrete track, (f) finished continuous concrete railway track.

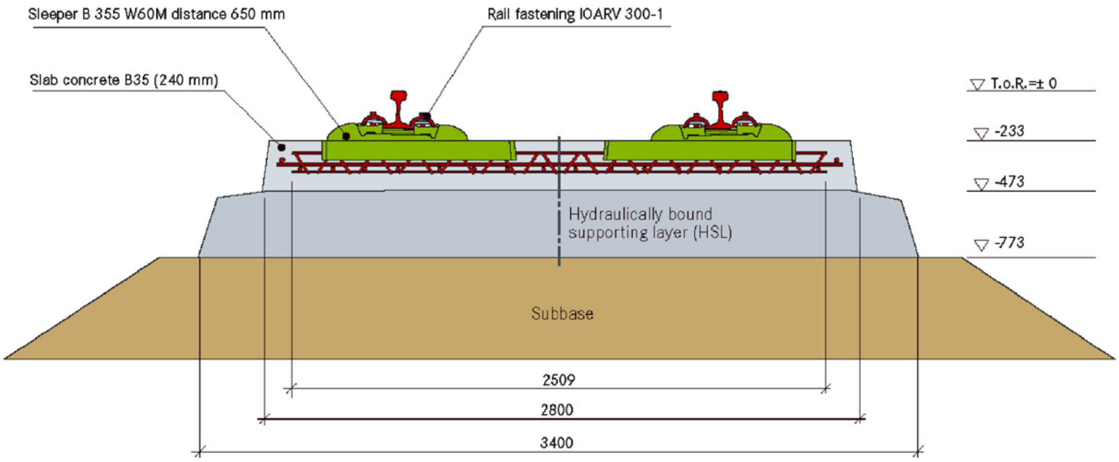

Fig. 2. Example of a slab track design with reinforcement in the neutral axis (Rheda 2000 system).

A more economic possibility to meet stiffness requirements is to increase the bending stiffness of the concrete slab-track itself, for example by applying an eccentric reinforcement. In the literature [4,5], a comparative analysis of both methods has been performed from a static point of view, including a fatigue analysis. The main objective of the present paper is to consider the question in a dynamic formulation. The aim is to minimize the amplitudes of the slab-track vibration, in view of a reduction of track deterioration. No standards, with which track vibrations must comply, exist. Therefore, the aim is just to minimize the amplitudes of the track vibration, which is equivalent to maximizing the dynamic track stiffness. The track is a component of the total train–track system. Therefore, maximizing the dynamic track stiffness may lead to, e.g. an increase of dynamic train-track interaction forces for running trains due to track irregularities. However, in general high dynamic interaction forces occur due to track irregularities with length-scales shorter than a bogie length (for longer length-scales, the vertical response may be assumed quasistatic). Further, the rail, which bears the running vehicle, is separated from the concrete slab via the fasteners and the railpads, which effectively isolate the slab from the rail in the short-wave excitation band. For these reasons the assumption that a stiffer slab will not lead to higher dynamic forces on the slab is reasonable, as well as the objective of slab vibration minimization.

In the context of track vibration mitigation, soil improvement under the embankment and embankment stiffening were studied earlier by Kaynia et al. [6] for a classical ballasted track on extremely soft soil, loaded by a constant moving train load. Andersen and Nielsen [7] considered both soil improvement and inclusion of a concrete box girder in the embankment, directly under the slab, as a countermeasure of track vibration. They included both load speed and frequency effects and found both measures to be effective at all frequencies and speeds. Also Sheng et al. [8] paid attention to the effect of the specific track properties on train-induced ground vibration. However, they considered a ballasted track, where the mass density of the track is the most important variable and not its bending stiffness.

2. Modelling the slab track railway: the dynamic stiffness of the track against arbitrary loading

The model which is used to investigate the dynamic response of a slab–track railway system to a running train axle is shown in Fig. 3, along with some notations.

The model consists of a beam on visco-elastic half-space subject to a moving load (Voight's material model is used for the half-space). According to recent investigations on slab-track by Savidis and Bergmann [9], realistic results can be obtained with the help of this basic model, i.e., model parameters can be adjusted in such a way that model predictions coincide with measurements. The beam cross-section is considered to be infinitely rigid, which is realistic for a slab track. The mathematical formulation of the model has been given in Ref. [10], where also the response of the beam and the half-space to moving constant and harmonic loads has been analysed.

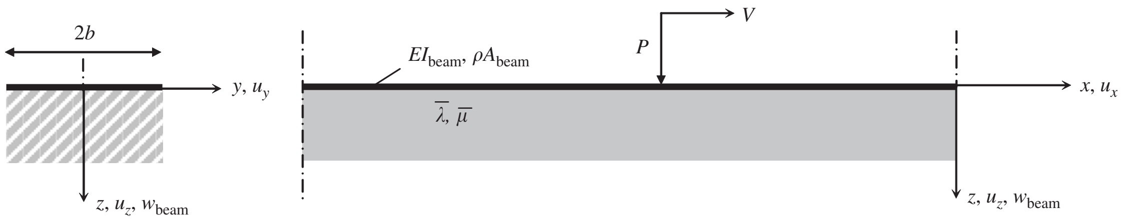

Fig. 3. Three-dimensional model for a slab track railway (vertical cross-sectional planes across and along the track axis).

It should be remarked that the spatial extension of a soil improvement is limited to the direct vicinity of the track, whereas in the half-space model only the stiffness of the complete half-space can be increased. Therefore, the applicability of the homogeneous half-space model for the soil depends on the stress distribution under the track. It was shown by Lansing in 1966 [11] that for a constant load moving at a subcritical velocity over a half-space, the eigenfield of the load is localized in the load vicinity and no wave radiation occurs. Thus, considering a constant load, it is not necessary to use more advanced soil models, for example introducing stratification. When a load spectrum is considered for a non-homogeneous soil, the error involved will increase with the contribution of radiated waves to the response. This contribution generally increases with the frequency and velocity of the load. Therefore, the model is applicable only for subcritical loading cases where the non-oscillating part of the loading prevails. However, especially for higher frequencies, whose wavelength is comparable to the depth and width of the soil improvement, the presence of the boundaries will affect the model predictions.

In the analysis in Ref. [10] the equivalent dynamic stiffness of the half-space [12] under the beam was introduced as

$$\chi = -\frac{1}{\tilde{\tilde{w}}_{\text{beam}}} \sum_{n=1}^{N} \tilde{\tilde{\mathbf{F}}}^{(n)}; \quad \boldsymbol{\chi}(k_x, \omega) = [\,\chi_x \quad \chi_y \quad \chi_z\,]^{\mathrm{T}} \tag{1}$$

in which $\mathbf{F}^{(n)}$ designates the interaction force vector between the beam and the half-space for an arbitrary strip n of the beam; n is a counter for the strips in the discretized beam—half-space interface ($1 \leqslant n \leqslant N$). The stiffness vector (1) describes the stiffness of the three-dimensional (3-D) half-space against the beam as a complex stiffness $\boldsymbol{\chi}$ of a 1-D elastic foundation, depending on the frequency and the wavenumber of flexural waves in the beam. The following equation was established in order to determine this stiffness:

$$\sum_{l=m-1}^{m-N} \mathbf{I}^{(l)} \boldsymbol{\chi}^{(m-l)} = \begin{bmatrix} 0 & 0 & -2\pi\mu\,\Delta y \end{bmatrix}^{\mathrm{T}}, \quad 1 \leqslant m \leqslant N \tag{2}$$

in which the integral matrices $\mathbf{I}^{(l)}$ were given by

$$I_{11}^{(l)} = -\mathrm{i}\frac{1}{k_x}\int_{-\infty}^{\infty} \frac{1}{\xi \bar{R}_s}\,\varepsilon\,\mathrm{d}\xi - \mathrm{i}\frac{1}{k_x}\int_{-\infty}^{\infty} \frac{4(1+\xi^2) - 3\beta_s^2 - 4\bar{R}_p\bar{R}_s}{\xi \bar{R}_s \bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \tag{3}$$

$$I_{22}^{(l)} = -\mathrm{i}\frac{1}{k_x}\int_{-\infty}^{\infty} \frac{1}{\xi \bar{R}_s}\,\varepsilon\,\mathrm{d}\xi - \mathrm{i}\frac{1}{k_x}\int_{-\infty}^{\infty} \xi\frac{4(1+\xi^2) - 3\beta_s^2 - 4\bar{R}_p\bar{R}_s}{\bar{R}_s \bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \tag{4}$$

$$I_{12}^{(l)} = I_{21}^{(l)} = -\mathrm{i}\frac{1}{k_x}\int_{-\infty}^{\infty} \frac{4(1+\xi^2) - 3\beta_s^2 - 4\bar{R}_p\bar{R}_s}{\bar{R}_s \bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \tag{5}$$

$$I_{13}^{(l)} = -I_{31}^{(l)} = \frac{1}{k_x}\int_{-\infty}^{\infty} \frac{2(1+\xi^2) - \beta_s^2 - 2\bar{R}_p\bar{R}_s}{\xi \bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \tag{6}$$

$$I_{23}^{(l)} = -I_{32}^{(l)} = \frac{1}{k_x}\int_{-\infty}^{\infty} \frac{2(1+\xi^2) - \beta_s^2 - 2\bar{R}_p\bar{R}_s}{\bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \tag{7}$$

$$I_{33}^{(l)} = -\mathrm{i}\beta_s^2 \frac{1}{k_x}\int_{-\infty}^{\infty} \frac{\bar{R}_p}{\xi\bar{\Delta}}\,\varepsilon\,\mathrm{d}\xi, \quad \text{where} \quad \varepsilon = \left(\mathrm{e}^{\mathrm{i}k_x\xi\,\Delta y(l-1/2)} - \mathrm{e}^{\mathrm{i}k_x\xi\,\Delta y(l+1/2)}\right). \tag{8}$$

The following notations were used:
$\xi = k_y/k_x$, $v_{\mathrm{ph}} = \omega/k_x$, $\beta_{p,s} = v_{\mathrm{ph}}/\tilde{c}_{p,s}$, $\bar{R}_{p,s} = R_{p,s}/k_x = \sqrt{1+\xi^2-\beta_{p,s}^2}$, $\bar{\gamma} = \gamma/k_x^2 = 2(1+\xi^2) - \beta_s^2$ and

$$\bar{\Delta} = \Delta/k_x^4 = 4\bar{R}_p\bar{R}_s(1+\xi^2) - (2(1+\xi^2) - \beta_s^2)^2. \tag{9}$$

In order to determine the total vertical stiffness of the track to a specified load, the beam equation must be introduced. For the loading a moving harmonic point load is chosen, representing a frequency component of the loading spectrum by a moving train axle (Fig. 2). The vertical motion of the beam is then described by

$$EI_{\mathrm{beam}} w_{\mathrm{beam}}(x,t)_{,xxxx} + \rho A_{\mathrm{beam}} w_{\mathrm{beam}}(x,t)_{,tt} - \int_{-b}^{b} \sigma_{zz}(x,y,0,t)\,\mathrm{d}y = -P\mathrm{e}^{\mathrm{i}\omega_0 t}\delta(x - Vt), \tag{10}$$

where ω_0 is the load frequency. The steady-state solution of this equation in the frequency–wavenumber domain is given by

$$\tilde{\tilde{w}}_{\mathrm{beam}}(k_x,\omega) = -2\pi P \frac{\delta(\omega_0 + \omega - Vk_x)}{D_{\mathrm{beam}}(k_x,\omega) + \chi_z(k_x,\omega)},$$
$$D_{\mathrm{beam}} = EI_{\mathrm{beam}}k_x^4 - \rho A_{\mathrm{beam}}\omega^2. \tag{11}$$

In the numerator, the chosen form of loading appears as the Dirac delta-function with the argument transformed to the frequency–wavenumber domain. Generally, the beam can be loaded by different types of loading, having different configurations in the spatial domain and different time dependences. Each loading on the beam may be transformed to the wavenumber–frequency domain. The stiffness of the beam–half-space system under this generalized loading is a function of both frequency and wavenumber. This stiffness appears in the denominator of Eq. (11) as a summation of the vertical dynamic stiffness of the half-space under the beam and that of the free beam. It will be referred to in this paper as a 'generalized dynamic track stiffness' K_z. It is a useful parameter to characterize the 'overall stiffness' of the track. K_z is given by

$$K_z(k_x,\omega) = D_{\mathrm{beam}} + \chi_z = EI_{\mathrm{beam}}k_x^4 - \rho A_{\mathrm{beam}}\omega^2 + \chi_z(k_x,\omega) \tag{12}$$

or, in terms of the phase speed of the waves propagating along the beam:

$$K_z(k_x,v_{\mathrm{ph}}) = EI_{\mathrm{beam}}k_x^4 - \rho A_{\mathrm{beam}}v_{\mathrm{ph}}^2 k_x^2 + \chi_z(k_x,v_{\mathrm{ph}}). \tag{13}$$

In order to perform a numerical analysis of K_z, the following representative parameter values are introduced:

- Beam (concrete slab) parameters:

$$2b = 3.20\,\mathrm{m},\ \ A = 1.10\,\mathrm{m}^2,\ \ I = 0.011\,\mathrm{m}^4,\ \ \rho = 2400\,\mathrm{kg\,m^{-3}}, E = 20\times10^9\,\mathrm{N\,m^{-2}},$$
$$\text{or } EI = 2.20\times10^8\,\mathrm{N\,m^2} \ \text{ and } \ \rho A = 2640\,\mathrm{kg\,m^{-1}}. \tag{14}$$

- Half-space (soil) parameters: $E_{\mathrm{soil}} = 8\times10^7\,\mathrm{N\,m^{-2}}$, $\upsilon = 0.3$, $\rho = 1960\,\mathrm{kg\,m^{-3}}$ (with these values the shear modulus G or μ equals $3\times10^7\,\mathrm{N\,m^{-2}}$). For the damping parameters γ_1 and γ_2 negligible values are chosen, so that solutions can be considered to be quasi-undamped:

$$\tilde{c}_p = c_p\sqrt{1-\gamma_1\mathrm{i}v_{\mathrm{ph}}};\quad \tilde{c}_s = c_s\sqrt{1-\gamma_2\mathrm{i}v_{\mathrm{ph}}};\quad \gamma_1 = 1\times10^{-7};\quad \gamma_2 = 1.5\times10^{-7}. \tag{15}$$

In Fig. 4, the real and imaginary parts of K_z are shown as functions of v_{ph}, for both short and long waves relative to the beam width. The graphs show the real part of the equivalent dynamic stiffness of the half-space,

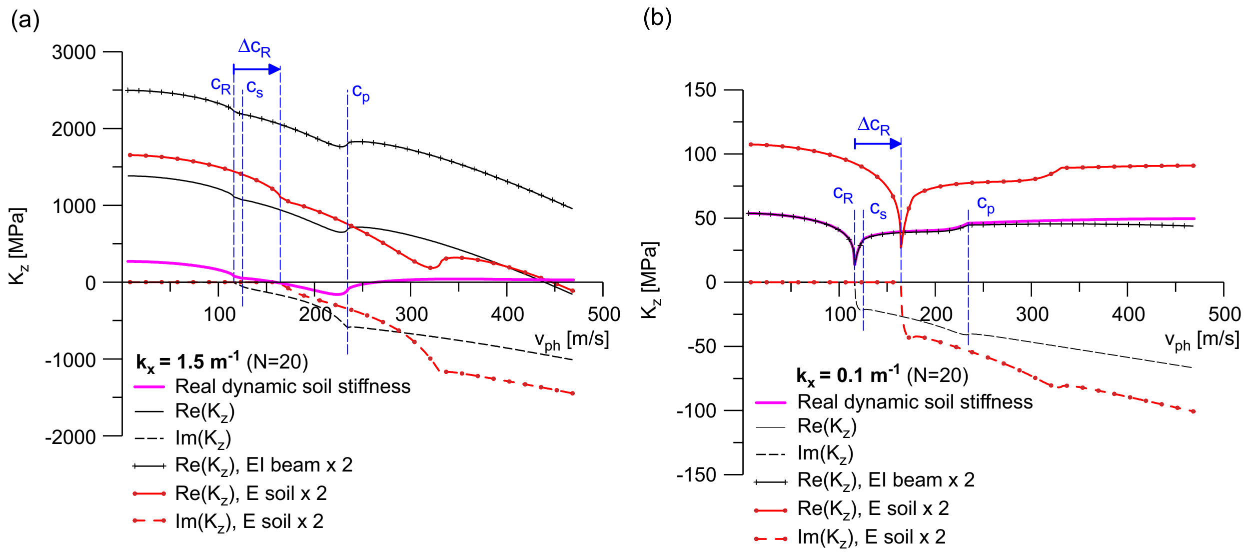

Fig. 4. Effects of an increase of the beam flexural stiffness and a soil improvement on the generalized dynamic track stiffness for short waves (a) and long waves (b).

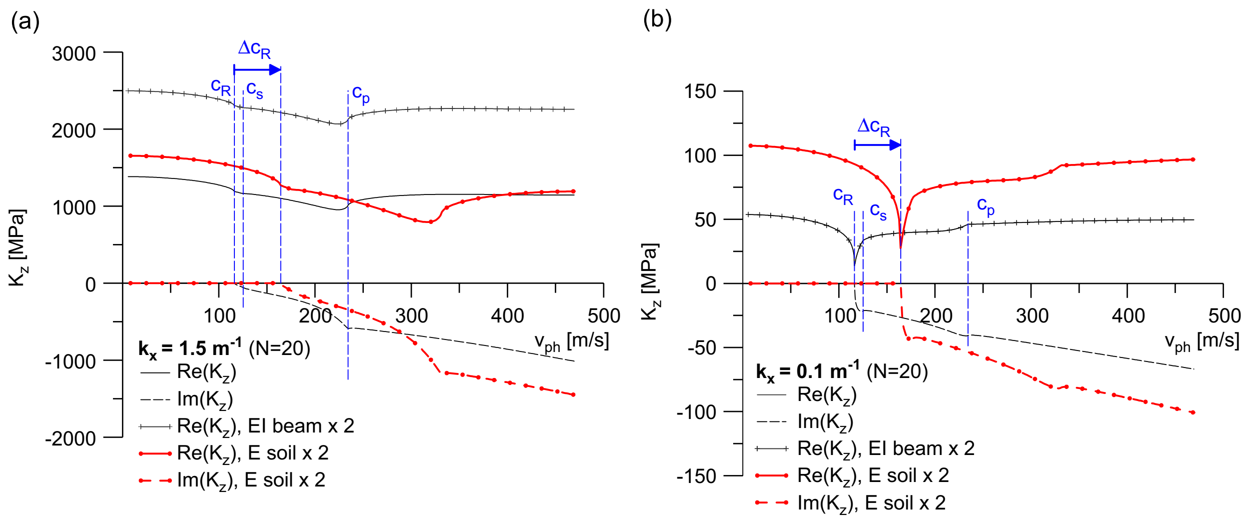

Fig. 5. Effects of an increase of the beam flexural stiffness and a soil improvement on the generalized dynamic track stiffness for short waves (a) and long waves (b) (as in Fig. 4), disregarding the mass of the beam.

the generalized track stiffness for a reference track according to the parameters (14) and (15), for a track with an increased beam stiffness (doubling of *EI*), and for a track on an improved soil (doubling of the Young's modulus).

In Fig. 5 the same graphs are shown as in Fig. 4, but assuming zero mass of the beam. Fig. 5 shows the effect of the distributed track mass on the dynamic track stiffness.

A number of conclusions can be drawn from Figs. 4 and 5 with respect to the dynamic track stiffness. They are discussed in the following. The dynamic track stiffness for short waves (high frequencies) is significantly higher than for long waves (low frequencies). This holds for both the real and imaginary parts of the stiffness, yielding also a higher radiation damping for high frequencies. Therefore, both radiation damping and material damping are most active in the high-frequency regime. Further, for short waves the contribution of the

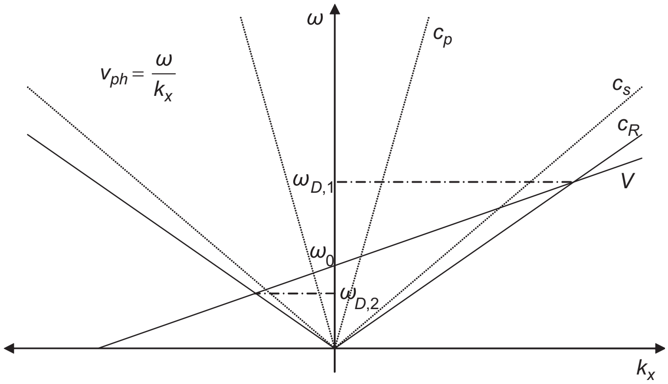

Fig. 6. Dispersion plane with Doppler effect for the beam–half-space system under a moving harmonic load at speed V (the influence of the beam on the dispersion of the system is disregarded).

dynamic beam stiffness to the total track stiffness is dominant with respect to the contribution of the dynamic soil stiffness. Therefore, for short waves beam stiffening is much more effective to reduce the track response than soil improvement. Again for short waves, the stiffness generally decreases with increasing frequency, and becomes negative in the high-frequency regime. This decreasing trend is due to inertia of the beam (as can be observed from Fig. 5a).

For long waves, the contribution of the dynamic soil stiffness to the total track stiffness is dominant with respect to the contribution of the dynamic beam stiffness, which is negligible. This implies that for low frequencies soil improvement is far more effective than beam stiffening.

The Rayleigh wave speed is, practically, a critical speed of the system. The corresponding dip in the stiffness (the stiffness is zero in the absence of damping) is much more pronounced for long waves. For short waves, this phase speed corresponds to a much higher frequency; therefore in this case the influence of the material damping is much larger. A notable increase of the critical velocity of the system can only be achieved by soil improvement.

A change in the moment of inertia of the beam has no effect on the imaginary part of the track stiffness, which is equal to the imaginary part of the dynamic soil stiffness (note that no material damping in the slab is accounted for). Therefore, radiation damping cannot be influenced without changing the soil properties. However, as load velocities are commonly below the critical speed, the radiation damping is not relevant for practice, when a constant load is considered. This is not true for the vibratory loading components, for which the radiation damping can play an important role also in the subcritical loading regime due to the Doppler effect and the corresponding frequency shift. According to Fig. 6, for vibratory loading the frequency of the waves in the track is related to the load velocity and its frequency by $\omega = Vk_x + \omega_0$ (see also Ref. [13] for the physical meaning of this relation).

3. Frequency response analysis of the track

In this section, the effect of the relative contributions of the dynamic stiffness of the soil and the slab under a frequency component of the load spectrum of a moving train axle on the track response will be investigated. As has been mentioned previously, the model presumes that the static load component prevails (which is generally the case for running trains). The harmonic load component is assumed to be of the form $P(t) = Pe^{i\omega_0 t}$. Subcritical load velocities ($V < c_R$) are considered. The inverse transform of Eq. (11) to the original x, t-domain yields

$$w_{\text{beam}}(x,t) = -\frac{P}{2\pi} e^{i\omega_0 t} \int_{-\infty}^{\infty} \frac{e^{ik_x(x-Vt)}}{D_{\text{beam}}(k_x, k_x V - \omega_0) + \chi_z(k_x, k_x V - \omega_0)} \, dk_x, \tag{16}$$

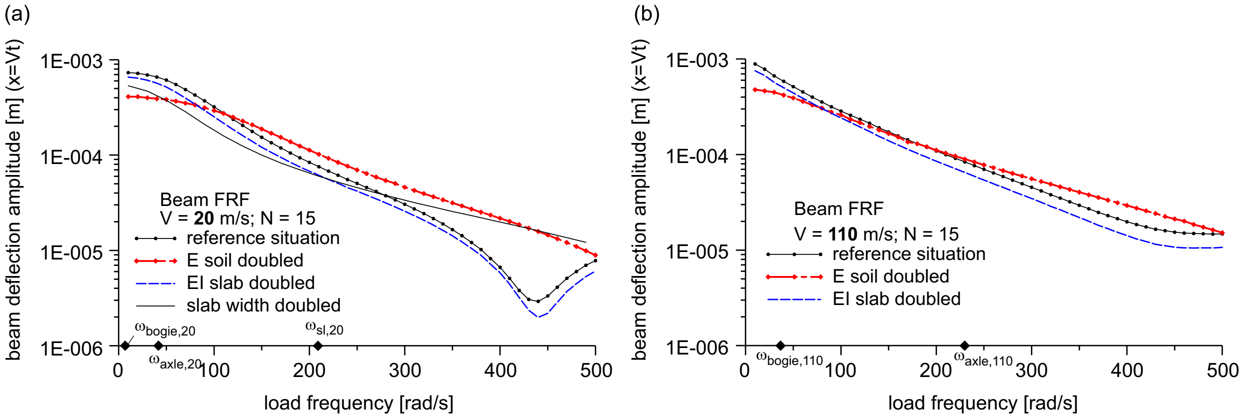

Fig. 7. Effects of an increased beam stiffness and a soil improvement on the slab frequency response at 20 m/s (72 km/h) and 110 m/s (396 km/h), respectively.

where the resulting displacement is a complex quantity. The displacement amplitude of the slab track under the travelling load is given by the absolute value of this function for $x = Vt$:

$$\left|w_{\text{beam}}(Vt, t)\right| = \frac{P}{2\pi}\left|\int_{-\infty}^{\infty} \frac{1}{D_{\text{beam}}(k_x, k_x V - \omega_0) + \chi_z(k_x, k_x V - \omega_0)}\, \mathrm{d}k_x\right|. \tag{17}$$

In Fig. 7 this amplitude is shown as a function of the loading frequency (10–500 rad/s or 1.6–80 Hz) for a track according to the parameters (14)–(15) and a track for which the soil Young's modulus and the beam stiffness have been doubled. The axle load has magnitude $P = 225$ kN and moves uniformly at velocities $V = 20$ m/s (72 km/h) and $V = 110$ m/s (396 km/h) along the track.

The graphs in Fig. 7 are similar, confirming that the velocity influence on the track frequency response in the subcritical regime is negligible (using the half-space model). This corresponds to the results of Bergmann [14], who found the influence of the train speed on bending moments in the slab to be very small as long as the train velocity does not approach its critical value.

Comparing the different frequency response functions (FRFs), it is observed that the FRFs for the improved soil have an intersection with the reference function. For low frequencies it shows a lower vibration level for the track than in the reference situation, whereas the performance for higher frequencies is worse. This can be explained by the effect of added mass (the component of soil reaction that is in phase with acceleration of the slab), which reduces the dynamic stiffness and increases displacements at high frequencies. The FRFs for an increased slab bending stiffness show a lower track vibration level for the whole frequency domain. The FRFs show no clear resonance peaks in the considered frequency domain. The small anti-resonance at 430 rad/s for 20 m/s is due to the width of the slab, as the results for the doubled slab width show in Fig. 7a. These results also show that the width of the slab is an effective parameter to reduce the slab displacements in the low-frequency regime, especially since this width can be increased without affecting the slab bending stiffness significantly. Further, for low frequencies the soil improvement is most effective, whereas in the high-frequency range the slab stiffening is far more effective, as was concluded before. Therefore, in general the whole load spectrum should be accounted for, in intelligent slab track design.

According to the measurements carried out with an ICE running on slab track as discussed by Nordborg [15], an important part of the frequency content of the track response spectrum will be situated around the sleeper passing frequency ω_{sl}, given as $\omega_{sl} = 2\pi V/d_{sl}$ (the sleeper distance d_{sl} is commonly 0.6 m). At 20 and 110 m/s follows: $\omega_{sl,20} = 209$ rad/s (33 Hz) and $\omega_{sl,110} = 1152$ rad/s (183 Hz). In Fig. 7 this frequency is indicated, but not at 110 m/s, as it is outside the considered frequency range, and the frequency response is already negligible at this frequency for the considered parameters.

According to the spectra of track vibrations as measured for a high-speed train on ballasted track and reported by Degrande and Schillemans [16] for a Thalys and by Auersch [17] for an ICE, two more frequencies are found to play a dominant role, namely the frequencies introduced by the periodical axle spacing and bogie

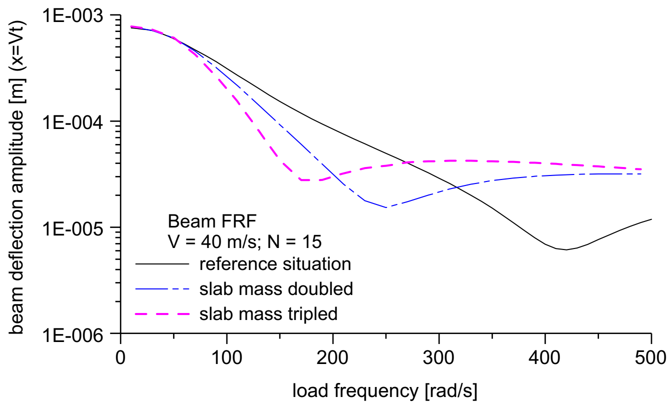

Fig. 8. Effects of an increased slab mass on the slab frequency response at 40 m/s (144 km/h).

spacing of the train vehicle. As these frequencies are the same for ballasted and non-ballasted track, also these values have been indicated for the respective velocities in Fig. 7. The axle passing frequency is given by $\omega_{\text{axle}} = 2\pi V/d_{\text{axle}}$ with $d_{\text{axle}} = 3\,\text{m}$ (Thalys). This yields $\omega_{\text{axle},20} = 42\,\text{rad/s}$ (6.7 Hz) and $\omega_{\text{axle},110} = 230\,\text{rad/s}$ (36.7 Hz). The bogie passing frequency is given by $\omega_{\text{bogie}} = 2\pi V/d_{\text{bogie}}$, where we choose $d_{\text{bogie}} = 18.7\,\text{m}$ (Thalys carriage), yielding $\omega_{\text{bogie},20} = 6.7\,\text{rad/s}$ (1.1 Hz) and $\omega_{\text{bogie},110} = 37\,\text{rad/s}$ (5.9 Hz).

From Fig. 7 it may be concluded that the response to the bogie passing frequency will be in general almost quasistatic, which is confirmed by the measurements [17] and earlier results [18] presented by Auersch. Therefore, to reduce the response at this frequency, soil improvement is a better solution. A general conclusion as to the axle passing frequency cannot be drawn, whereas for reduction of the response at the sleeper passing frequency an increase of the track stiffness is most effective.

In the following, the effect of the distributed mass of the slab is considered. Normally, a change in the distributed mass is obtained by a change in the cross-sectional area. The latter change implies also a change of the slab bending stiffness. However, as there is no unique relation between the increase of the slab cross-sectional area and the moment of inertia (the shape should be taken into account), the slab mass is treated as an independent variable for analysis of its influence. It can be shown that an additional increased slab stiffness reduces the beam frequency response at all frequencies.

The effect of an increase of the slab mass ($\rho A_{\text{beam}} \to 2\rho A_{\text{beam}}$; $\rho A_{\text{beam}} \to 3\rho A_{\text{beam}}$) is shown in Fig. 8, for a load velocity of 40 m/s. The calculated FRFs have two intersections with the reference function; clearly an anti-resonance is related to the slab mass and shifts towards lower frequencies with increasing slab mass. For low frequencies (close to 'statics'), the effect of the inertia of the slab is very small, as can be expected. In frequency regime ranging from 10 to about 50 Hz a higher slab mass can reduce the slab response significantly, whereas for relatively high frequencies ($>50\,\text{Hz}$) the increased slab mass increases the track response. The increase can be explained by the reducing effect of added mass on the dynamic track stiffness (Figs. 4 and 5). An unambiguous conclusion with respect to the effect of the track mass on its response cannot be drawn and depends both on the actual track parameters and the load spectrum under consideration. However, comparing Fig. 8 to Fig. 7 (right), it can be concluded that an increase of the track mass is generally advantageous in the most relevant part of the frequency domain, for trains running at subcritical speeds.

4. Related aspects in railway track design

There are a number of aspects which should be considered in the design of a slab track railway and related to the dynamic behaviour of the train–track system. These include the excitation of environmental vibrations, comfort requirements for passengers and vehicle ride quality, and long-term track behaviour in terms of deterioration. Each of them will be addressed briefly regarding their relation to the dynamic track stiffness in the following.

It can be shown that for the constant part of the load the effect of slab stiffening on the level of environmental vibrations is negligible. The amplitude spectrum decreases slightly only in the high-frequency regime, where it has been shown that also the slab deflections decrease. With respect to the oscillating part of the load the same effect can be observed.

Comfort requirements (vehicle ride and passenger comfort) are mostly related to long wave components of irregularities in the track and low-frequent oscillations, most of the energy in the higher-frequency vibrations being absorbed efficiently in the components of the track (pads) and train vehicles (dampers). In order to predict the effect of a stiffer track on the comfort levels, the response of the total train–track system to these long-wave irregularities must be investigated.

Track deterioration is mostly related to the magnitude of train–track interaction forces, which mainly occur due to short-wave components of track irregularities. As has been pointed out in the introduction, the slab is largely isolated from the rail in the short-wave excitation band. Therefore, given the same roughness spectrum of the track, it is difficult to predict the effect of a stiffer track structure on the level of interaction forces.

5. Conclusions

Soil improvement and increasing the slab bending stiffness have been considered as possibilities to meet stiffness requirements for slab track high-speed railways. The track and the representative train loading have been modelled as a beam on visco-elastic half-space subject to a moving load. The generalized vertical dynamic track stiffness against an arbitrary loading in the frequency–wavenumber domain has been introduced, and compared for both methods. It has been found that for high frequencies an increase of the slab stiffness is most effective, whereas for low frequencies soil improvement is a better solution. The latter also has the advantage of increasing the critical train velocity. An increase of the slab–soil contact width reduces the slab displacements in the low-frequency regime; as this width can be increased without changing the slab stiffness significantly, a large width is always advantageous.

An increase of the track mass generally results in lower track displacements in the frequency regime, which is of interest for trains running at subcritical speeds.

Summarizing, as a most economic solution one may think of requirements of a minimum stiffness of the subsoil in relation to train passenger comfort and critical speeds, whereas the remaining part of the required stiffness can be provided by the shape-optimized slab with a maximum contact width.

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